Creativity in mathematical learning: A model to explain the cognitive functioning of creative processes and provide general design requirements

Creatività nell’apprendimento della matematica: Un modello per spiegare il funzionamento cognitivo dei processi creativi e fornire requisiti generali di task design

Creatividad en el aprendizaje de las matemáticas: Un modelo para explicar el funcionamiento cognitivo de los procesos creativos y proporcionar requisitos generales de diseño de tareas

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Abstract. *The interest in creativity in mathematics education has increased in the last few years at all levels. Nevertheless, a comprehensive framework for creativity in mathematics education that supports the design of appropriate tasks is still missing. In this paper ta connection between different elements coming from mathematics education and psychology is built to provide a theoretical model for the explanation of the cognitive functioning of creative processes. For this purpose, the creativity characteristics of originality and flexibility are used to categorize creative processes, then the defined categories are cognitively characterized by elements of conceptual blending theory. Finally, general requirements of task design to support creativity in mathematics education are developed, consistently with the introduced model. The model can be used for ‘navigating’ the existing literature and identifying redundancies and synergies between different theoretical approaches; it can also be intended as a tool for detecting and classifying instances of creativity. Moreover, the introduced design requirements allow to work in the direction of task design to stimulate and foster creativity in mathematics education.*

Keywords: mathematics education; creativity; task design; cognitive model; design requirements

Sunto. *Negli ultimi anni, l'interesse per la creatività in didattica della matematica è aumentato a tutti i livelli. Tuttavia, manca ancora un framework completo per la creatività in didattica della matematica che supporti la progettazione di task appropriati. In questo articolo viene stabilita una connessione tra diversi elementi provenienti dalla didattica della matematica e dalla psicologia, al fine di fornire un modello teorico in grado di spiegare il funzionamento cognitivo dei processi creativi. A tal fine, due caratteristiche della creatività, che sono l'originalità e la flessibilità, vengono utilizzate per categorizzare i processi creativi; le categorie definite vengono caratterizzate cognitivamente mediante elementi della teoria di conceptual blending. Infine, vengono sviluppati dei requisiti generali per la progettazione delle attività con l'obiettivo di sostenere la creatività in didattica della matematica in coerenza con il modello introdotto. Tale modello può essere utilizzato per 'navigare' la letteratura esistente e identificare ridondanze e sinergie tra diversi approcci teorici; esso può essere anche inteso come strumento utile per individuare e classificare istanze di creatività. Inoltre, i requisiti di progettazione introdotti consentono di lavorare nella direzione della progettazione di task utili per stimolare e favorire la creatività in didattica della matematica.*

Parole chiave: educazione matematica; creatività; progettazione didattica; modello cognitivo; requisiti di task design

Resumen. *En los últimos años, el interés por la creatividad en la enseñanza de las matemáticas ha aumentado a todos los niveles. Sin embargo, sigue faltando un marco global para la creatividad en la educación matemática que respalde un diseño adecuado de las tareas. En este artículo se establece una conexión entre distintos elementos de la educación matemática y la psicología con el fin de proporcionar un modelo teórico que pueda explicar el funcionamiento cognitivo de los procesos creativos. Con este fin, se utilizan dos características de la creatividad, que son la originalidad y la flexibilidad, para categorizar los procesos creativos; las categorías definidas se caracterizan cognitivamente utilizando elementos de la teoría de la conceptual blending. Por último, se desarrollan requisitos generales para el diseño de actividades con el objetivo de apoyar la creatividad en la educación matemática de forma coherente con el modelo introducido. Este modelo puede utilizarse para 'navegar' en la literatura existente e identificar redundancias y sinergias entre diferentes enfoques teóricos; también puede entenderse como una herramienta útil para identificar y clasificar instancias de creatividad. Además, los requisitos de diseño introducidos permiten trabajar en la dirección de diseñar tareas útiles para estimular y fomentar la creatividad en la educación matemática.*

Palabras clave: educación matemática; creatividad; diseño didáctico; modelo cognitivo; requisitos de diseño de tareas

1. Introduction

There are at least two arguments in favor of the importance of creativity in mathematics education. Firstly, creativity is recognized as an important twenty-first century skill (Pellegrino & Hilton, 2012) necessary to face the exponential growth of innovation in all areas of life associated with technological progress and scientific advancement (Leikin & Sriraman, 2021). Secondly, creativity is a necessary condition for insightful and lasting learning, as it is an essential ingredient in problem solving processes (Haavold & Sriraman, 2022; Leikin & Elgrably, 2022; Schoevers et al., 2022), but also in argumentation processes, intended as a sequence of original and plausible arguments that are based on mathematical properties (Haavold et al., 2020; Lithner, 2008). Indeed, each authentic problem-solving activity, focused on a problematic situation never faced before, requires a certain kind of creative thinking.

Summing up, creativity in mathematics education is crucial not only for its practical value in preparing students for future scientific challenges but also as an integral part of the discipline itself, as its primary importance lies in fostering deep and lasting learning, essential for navigating future technological advancements. Therefore, creativity's role in mathematics should be valued for its own merit, beyond its specific practical applications. But how can creativity be stimulated in the context of the 'every day'-classroom-practice, which kind of tasks could be used and why are they appropriate in this sense?

While till now in mathematics education research an important effort was made to design suitable tasks for particular experimental settings, with the aim to produce diagnostic tools able to capture creativity instances (e.g., Leikin, 2013; Schindler & Lilienthal, 2022; Singer & Voica, 2022), general design requirements[1] for tasks that would allow to stimulate creativity are still underinvestigated. Such design requirements would greatly benefit research in mathematics education by enhancing our understanding of how to stimulate and develop creativity. They would also serve as valuable tools in teacher training, assisting educators in systematically designing suitable tasks. However, these task design requirements must be theoretically well founded on cognitive principles. This grounding is essential to explain how and why these requirements are crucial for designing tasks able to foster and potentially develop instances of creativity. Indeed, without a link of the design requirements to the cognitive tools, creativity stimulating tasks appear as a sort of 'black boxes' that transform non-creative students into creative ones. This paper presents a model

[1] We prefer to use the term 'design requirements' rather than 'design principles' because we are focusing on deducing such requirements theoretically, rather than inductively by abstracting from successful examples (Bell et al., 2004; McKenney & Reeves, 2018).

designed to generate these general design requirements. It integrates theoretical concepts from both mathematics education and psychology, with the goal of identifying design requirements tailored specifically to mathematics education.

2. State of the art, research problem and research questions

In this section, we first focus on the literature regarding the criteria for task design that promote creative thinking in mathematics education and then characterize the research problem and formulate the research questions.

2.1. State of the art

In their recent survey on creativity in mathematics, Leikin and Sriraman (2022) scrutinize 25 papers categorized under the title “Creativity related to practices and mathematical tasks.” While all the papers of this category discuss tasks that may stimulate creativity, only a handful offer some arguments advancing the discussion on task design requirements. In the following we shortly discuss the five papers that are closely related to this topic, pointing out their relation to our focus on general task design requirements.

- Levav-Waynberg and Leikin (2012) produce a longitudinal study that compares knowledge and creativity development between experimental and control groups using student-written tests. The research focuses on certain creativity criteria, like fluency and flexibility, suggesting that they are more amenable to enhancement and thus more ‘teachable’ than other criteria, such as originality. This study sheds light on how task design might bolster creativity, but it also helps to clarify which aspects of creativity can be developed ‘on a large scale’ and which may need more targeted strategies to reach more students. However, the paper’s emphasis is on the application of specific tasks rather than on establishing broad criteria for task design.
- Lee (2017) provides a diagnostic model for the graduation of teachers’ ability to construct tasks suitable for creativity education. This approach is helpful to better understand how teacher can act on higher or lower levels in designing tasks to foster creativity, but it does not provide general requirements for task-design.
- Boesen et al. (2010) examine how different tasks affect students’ mathematical reasoning during national tests. They find that fostering creativity requires challenging tasks that diverge from routine classroom activities and push students beyond their comfort zones. However, the study’s purpose is primarily diagnostic and does not focus on establishing universal task design requirements.

- Aljarrah (2020) explores collective creativity in mathematical learning, identifying four key creative acts within groups: summing forces, expanding possibilities, divergent thinking, and assembling things in new ways. While the study highlights cognitive tools that support creativity, it does not connect them to task design criteria, which would be relevant for research on general task design requirements.
- Sriraman and Dickman (2017) propose a reflection on tasks that require insight into mathematical pathologies, that means examples of counterintuitive behavior, showing how they can support creativity in the classroom. This paper focuses on the characteristics that make mathematical problems suitable for creativity education, but it does not focus on the criteria to be used to support task design in general and does not explain how they are related to the emergence of creativity.

2.2. Research problem and research questions

While most of the contributions discussed in section 2.1. offer structured tasks to enhance creative thinking, possibly including some general design requirements, none of them entirely elucidate the assumption about the efficacy of such tasks in nurturing creativity in mathematics education. This is not at all surprising since the rationale of the articles is not focused on such general aspects, but if we look at the related literature from this perspective, there is a lack of a cohesive and thorough framework able to support understanding of creative processes and their cognitive underpinnings. Indeed, each of the presented papers introduces a set of tools or operational criteria specific to the problem faced in the research, but none provides a theoretical synthesis of such tools or operational criteria. This paper aims to reduce the heterogeneity and fragmentation of the scenario, marked by a variety of design solutions, partly alternative and partly redundant or overlapping, by providing an overarching framework. For this purpose, we explore the link between cognitive mechanisms and task design, supposing to significantly clarify in this way the characteristics of the methods and the means to enhance creativity in mathematics learning. To achieve this, we first aim to establish a theoretical basis for the emergence of creative features and then elucidate their cognitive functioning. The cognitive tools that support this cognitive functioning and foster creative thinking are supposed to bridge the gap between student's creativity, as reported in the literature, and the requirements of task design intended to stimulate it.

The research question that emerges from the research problem characterized above is thus the following:

Which are general requirements of task design for tasks able to stimulate and *foster creativity in learning mathematics?*

Starting from what was previously mentioned about the limitations of current research on task design, this research question is divided into two sub-questions:

a) *Which are the characteristics of a theoretical model able to provide a synthetic and comprehensive explanation of the cognitive functioning of creative processes?*
b) *Which task-design requirements can be deduced from this model in order to stimulate and foster creativity in mathematical learning?*

3. Theoretical Framework

In this section, we present the theoretical frameworks we have drawn upon to answer the first research sub-question. In Section 3.1., we provide a preliminary clarification on the specific conception of creativity we focus on in this paper. In Section 3.2., we introduce the theory of conceptual blending, drawn from psychology, which serves as the primary theoretical reference to anchor the construction of our model for the explanation of the cognitive functioning of creative processes.

3.1. A suitable characterization of creativity

In mathematics education research, there is no universally agreed definition of creativity. Rather, there are common theoretical references where dimensions, types and characteristics of creativity are distinguished, and from which each study develops its own specific definition. In this section we clarify which *dimensions, types* and *characteristics of creativity* are relevant to our research purposes, and we present the concept of creativity we have chosen to work with.

1) *Dimensions*. With regard to the different dimensions of creativity, we consider Rhodes' (1961) work on the 4 P's of creativity, widely shared in mathematics education (e.g., Pitta-Pantazzi et al., 2018): creativity seen as a feature of either a *person* (understanding the traits, characteristics or attributes of the creative person), a *process* (describing the operations or stages of thinking used in the creative process), a *press* (examining the nature of situations and its context within the creative press), or a *product* (identifying outcomes and qualities of creative products). As clarified in section 2, in this paper we aim to provide an explanation of the cognitive functioning of creative processes. Accordingly, we focus our attention on the *process*, that is on creativity as a way of thinking, and do not consider

explicitly the other three dimensions[2].

2) *Types*. Regarding the types of creativity, we refer to the 4c model (Kaufman & Beghetto, 2009). These authors identified four types of creativity: (a) Big-Creativity (*Big-C*) exhibited by individuals who have won prestigious prizes and have gained long-term recognition; (b) Pro-creativity (*Pro-c*) or expert creativity; (c) Little-creativity (*little-c*) or ordinary creativity, manifested in everyday activities and discernible to others; and (d) Mini-creativity (*mini-c*), which is defined as the “novel and personally meaningful interpretation of experiences, actions and events” and is “involved in the construction of personal knowledge and understanding” (Beghetto & Kaufman, 2007, p. 73). In this paper, we aim to clarify the distinctive features of creative thinking processes that underpin personal learning in mathematics. Therefore, we focus exclusively on the ‘mini-c’ type of creativity.
3) *Characteristics of creativity*. Concerning the characteristics of creativity, we consider the psychometric models that have informed research on creativity in mathematics education (Joklitschke, et al., 2022). More specifically, we focus on two of the three criteria identified by Guilford (1959): *flexibility* (the number of different categories of solutions to the same problem) and *originality*[3] (the unusualness of solutions), while we do not consider fluency (the number of solutions) because this feature relates to the assessment of creative performance, which is beyond the scope of this article. Originally, the concepts of flexibility and originality were introduced to study divergent thinking and to define variables useful for the experimental measurement of a person’s creative abilities. Rather than viewing them as indicators for the empirical observation of divergent thinking performances, we regard these characteristics of creativity as conceptual categories to describe creative processes in a broad sense.

Holding together the premises on *dimensions, types, and characteristics of*

[2] By this we do not mean that creativity can be reduced to cognitive aspects alone and/or that the process is separable from the other dimensions or is more important than them. Ours is merely a choice of analysis that follows from the research questions we are working on. Indeed, as Rhodes himself points out: “each strand has unique identity academically, but only in unity do the four strands operate functionally” (Rhodes, 1961, p. 307).

[3] Torrance’s work (1974), which is also relevant in mathematics education, distinguishes similar characteristics of creativity, replacing the term ‘novelty’ with ‘originality’. We consider these two terms interchangeable. However, as mentioned above, in this work we focus on the process dimension. Therefore, like flexibility, the characteristic of originality/novelty must also be understood in relation to this dimension. Thus, we do not speak of originality in absolute terms, but only in relation to thought processes that are unprecedented for the subject performing them.

creativity, in this study we focus on originality and flexibility as distinctive properties of the mini-c processes that inform mathematical learning.

Our goal is to explain the cognitive functioning of creative thinking processes that enable students to use prior knowledge in new ways and generate novel solutions, ideas, and strategies.

Creativity, as defined in our research, has been deeply discussed in mathematics education. Pitta-Pantazzi and colleagues (2022) highlight three essential forms of ‘mini-c’ creativity: (a) building intuition and abstractness of a mathematical concept; (b) creating, manipulating, and connecting representations; (c) expressing flexible thinking. We take these instances as a point of reference, in accordance with the concluding remark of the quoted study, not because they “present an exhaustive list,” but because they “have the potential to capture mini-c related to various mathematical concepts (arithmetic, algebraic, geometric, statistical and measurement)” (Pitta-Pantazzi et al., 2022, p. 65). Indeed, the mini-c classification provides criteria that shed light on the cognitive requirements necessary for creativity tasks in mathematics education. In section 4, we will revisit these three ‘mini-c’-instances, introducing them a in our model that connects them to the overarching characteristics of ‘mini-c’ processes and presents a systematic classification of creative processes grounded in their cognitive functioning.

3.2. Conceptual blending

In section 2 we highlighted a gap in mathematics education research: the lack of a detailed cognitive explanation for how instructional methods and tasks promote creativity. Existing requirements for task design propose ways to encourage original and flexible thinking, yet they do not adequately explain the cognitive functioning of creative processes. Consequently, these requirements provide assorted solutions but lack an overarching structure.

To tackle the issue of providing such an overarching structure, we turn to Fauconnier and Turner’s theory on conceptual blending (Fauconnier & Turner, 2002; 2003). According to these authors:

> Conceptual blending is a basic mental operation that leads to new meaning, global insight, and conceptual compressions useful for memory and manipulation of otherwise diffuse ranges of meaning. It plays a fundamental role in the construction of meaning in everyday life, in the arts and sciences, and especially in the social and behavioral sciences. The essence of the operation is to construct a partial match between two input mental spaces, to project selectively from those inputs into a novel ‘blended’ mental space, which then dynamically develops emergent structure. (Facounnier & Turner, 2002, p. 2)

As we will explain in the following, creative thinking is one of the human processes that the theory of conceptual blending can help to clarify and characterize from a cognitive point of view. But let us first deepen some concepts involved in the definition of conceptual blending. A key concept referenced in the definition that requires further explanation is the one of 'mental space,' which is crucial in this context:

> Mental spaces are small conceptual packets constructed as we think and talk, for purposes of local understanding and action - they are very partial assemblies containing elements, structured by frames and cognitive models. (Facounnier & Turner, 2002, p. 2)

The idea of mental space is general in scope and in the context of this paper is to be understood as a cognitive framework in which different elements are related within ordered structures to organize knowledge and to gain explanatory and predictive control over experience. A precise set of rules, more or less explicit, informs each of these mental spaces, establishing which elements, which relations and which structural configurations are allowed within them. Examples of mental spaces which are also relevant to the exercise of creativity in learning mathematics are the scripts (Abelson, 1981). Scripts can be intended as anticipatory procedural schemas for the organization of ordinary situations, used to guide actions and strategic choices within a given context (D'Amore, 1999).

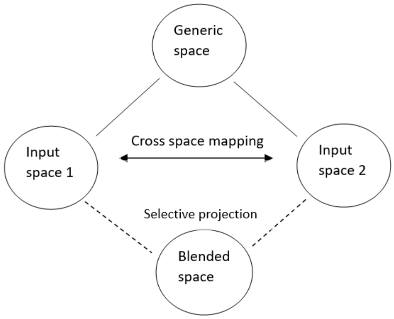
Conceptual blending operations are characterized by an interaction between two or more mental spaces:

In its most basic form, a conceptual integration network consists of four connected mental spaces: two partially corresponding input spaces, a generic space consisting of structures common to the inputs, and a blended space. (Facounnier & Turner, 2002, p. 4)

The basic structure of a conceptual blend is represented in Figure 1.

Figure 1

The basic structure of conceptual blending



The generic space allows homologies and correspondences to emerge between the two mental spaces, making their comparison and/or integration possible. In the blended space, on the other hand, the elements of the two input spaces are projected to bring forth an emergent novel space, with its own set of rules, elements, relations and structures:

> Projection allows the emergent structure to develop on the basis of composition (blending can compose elements from the input spaces to provide relationships that do not exist in the separate inputs), pattern completion (based on background patterns that are brought into the blend unconsciously), and elaboration (treating the blend as a simulation and 'performing' it imaginatively). (Facounnier & Turner, 2002, p. 4)

The blending process between two input spaces can occur in various ways, categorized into different types of "blending networks" (Facounnier & Turner, 2002, p. 6): *simplex networks*, *single-scope* and *double-scope networks*. In the following we characterize these blending networks.

In *simplexes networks*, input space 1 consists of a frame, which is a conventional, schematic organization of knowledge, and input space 2 consists only of specific elements that are assimilated within inputs space 1. By assimilation, we mean the process that allows the acquisition of new data using pre-existing mental frameworks or structures. In simplex networks, new external data from input space 2 are interpreted within the constituent rules of input space 1 and are thus integrated in it.

In *double-scope* and *single-scope networks*, both input space 1 and input space 2 consist of a frame. In *double-scope networks*, input space 1 and input space 2 are brought together, and they both contribute to the final organizational frame of the blended space. In *single-scope networks*, instead, the blended space inherits only the organizational frame of one of the two input spaces.

In section 4.1 we provide examples and a more detailed exploration of the blending networks simplex, single-scope, and double-scope. Our purpose is to utilize them to explain the cognitive functioning of creative processes involved in mathematical learning.

4. The Cognitive Creative Model and task design requirements

In this section, in response to the first research sub-question, we present a model to explain the creative functioning of creative processes.

The development of the model occurs in two phases. Firstly, in Section 4.1., we delve into the characteristics of originality and flexibility commonly associated with creative thinking. We define three distinct types of creative processes, each embodying originality and/or flexibility in varied ways. To relate these broad categories to mathematics education specifically, we link them with the ‘mini-c’ instances highlighted by Pitta-Pantazzi and colleagues (2022). Secondly, in Section 4.2., we advance to the explanatory phase, applying conceptual blending theory to clarify the cognitive functioning of the creative process categories established in 4.1.

4.1. The categories of creative processes

In this section we discuss the categorization of creative processes in the context of mathematics education, emphasizing the role of originality and flexibility as key characteristics of such processes.

We first use the characteristic of originality to distinguish between creative and non-creative processes. Subsequently, we introduce three different categories of creative processes, in which the property of originality, which they all share, relates differently to that of flexibility.

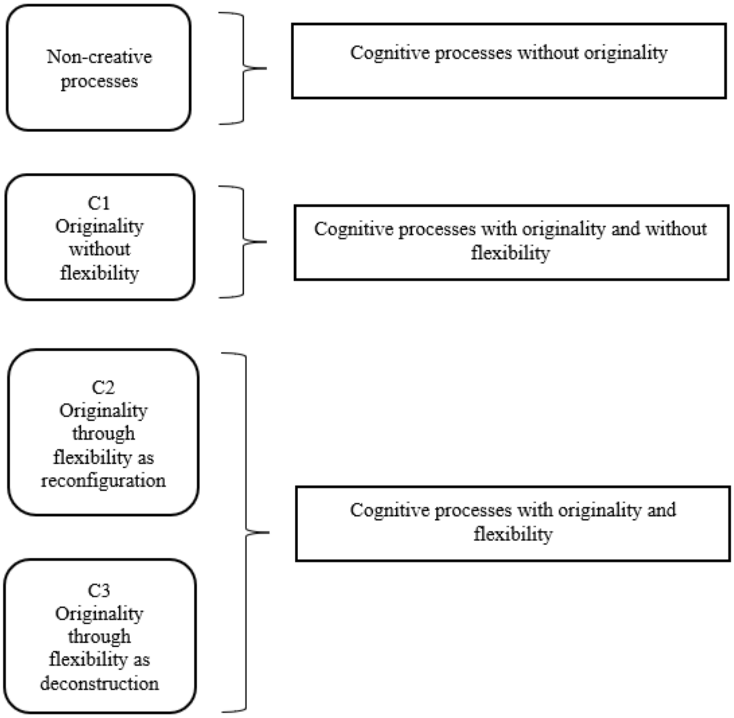
Originality is essential for a process to be considered creative. It involves elements of discovery and introduces new ideas, as opposed to reproductive thinking, which is about recalling or repeating existing knowledge (Lithner, 2008). For example, in mathematics education, solving problems requiring new strategies is seen as creative, while merely reproducing known procedures is not (Asenova et al., 2022; Zan, 1998).

In this sense, in our model the characteristic of originality is considered as a feature of each of the different types of creative processes.

The property of flexibility, on the other hand, is optional. Flexibility, when combined with originality, allows for the differentiation of three types of creative processes that we indicate as C1, C2, C3 (Figure 2).

Figure 2

The first step of the model: different categories of creative processes



C1 does not involve flexibility; C2 and C3 both connect originality to flexibility but involve flexibility in two different ways from a cognitive perspective. In the following we expose the three categories of creative processes, C1, C2 and C3, illustrating them by examples taken from the study of Pitta-Pantazzi et al. (2022) on mini-c instances in mathematics education.

1) *The first category C1* involves original thinking without flexibility. It is about logically combining known elements to discover new solutions or reasoning methods, maintaining their original properties. In mathematics education, instances of "involving insights of mathematical concepts" (Pitta-Pantazzi et al., 2022, p. 57), can fall into this category. In the specific example cited in the article, sixth-grade students encounter the concept of arithmetic mean through a realistic scenario in which various critics provide numerical scores for films. When faced with the problem of judging and comparing films based

on these numerical ratings, some students exhibited examples of processes akin to C1, reasoning as follows: if numbers quantify ratings, then it is reasonable to assume that sets of these numbers can be compared with each other to generate broader and more comprehensive evaluations. In this way, just by logically extending the premises of the initial situation, students creatively "exhibited understanding of the mean concept as a number that is representative of a set of numbers" (Pitta-Pantazzi et al., 2022, p. 57), which they had never met before.

2) *The second category C2* of creative processes integrates the property of originality with that of flexibility. In these cases, flexibility means that what is already known is not simply developed in continuity with its ordinary premises but is rethought from a new perspective.
   a. *Originality* emerges through operations of comparison, connection, and combination between elements that are generally not related and which, as a result, are reconfigured in their fundamental features. Several cases of mini-c that have emerged from research in mathematics education fall into this typology. Picking up on the previous example, students tackled the problem of film reviews not only by reasoning from the simple comparison of numerical grades but also by answering questions and using digital tools centered on the analogy between the calculation of the arithmetic mean and the visual operations of balancing weights on a seesaw. This is an example of mathematical insight in which flexible thinking supports creative reasoning. The discovery of a new function for numbers is developed around an analogical correspondence between usually distant and unrelated domains of experience, such as rating movies and balancing weights on a seesaw. In this case, moreover, the analogy is not only functional to an initial intuition of the concept of arithmetic mean but also supports a subsequent "Precision phase" (Pitta-Pantazzi et al., p. 54) in which students propose arguments and invent calculation strategies by developing the logical consequences of the analogy. The example is significant in clarifying how this type of creative process relates to another instance of mini-c relevant to mathematics teaching and learning, called "creation, manipulation, and connection of representations" (Pitta-Pantazzi et al., 2022, p. 60).
   b. *Flexibility* of the creative process C2 is expressed in the ability to identify innovative forms of knowledge representation and to work on semiotic transformations - treatments within the same semiotic register and conversions between two different semiotic registers (Duval, 1995; 2011/2017) – that are usually kept distinct and unconnected. In the

specific case examined here, against the backdrop of the heuristic analogy with balancing, the reasoning and calculations developed by the students are not confined to a single register but integrate different forms of representation: "These instances emerged as students were (i) exploring the different representations of the applet; (ii) switching between the pictorial digital balance representation and its verbal interpretation; (iii) recreating the balance model on their worksheets; and (iv) using the pictorial representation on their worksheets as a tool to find the mean" (Pitta-Pantazzi et al., 2022, p. 60).

3) *The third category C3* of creative processes is characterized by a different meaning of flexibility as the one used above. Flexibility is here intended as the ability to suspend and deconstruct established thinking structures to open up new creative possibilities. These creative processes are often linearly linked to the other two categories because they provide a premise or are recursively activated on already elaborated creative solutions to find alternatives. In mathematics education, the mini-c instance classified as "flexible thinking" (Pitta-Pantazzi et al., 2022, p. 62) best exemplifies this third category. The students in the study under consideration showed cognitive flexibility not only in the creative solutions with which they interpreted and represented the scores and their balancing but also in their ability to alternate and integrate different calculation strategies. Their strategies were based on the visual seesaw model and on the classical algorithm introduced to them by the teacher, without either of the two constituting a rigid constraint that prevented them from considering and valuing the other. Moreover, even within the same semiotic register and the same model or procedure, this type of cognitive flexibility emerged when students proposed different sets of numbers for the same mean and different strategies for identifying them (Pitta-Pantazzi et al., 2022).

So far, we introduced a classification in which the psychometric properties of originality and flexibility were reinterpreted as indicators to distinguish different kinds of cognitive functioning of creative processes and were explicitly linked to the mini-c instances identified in mathematics education. These different kinds of cognitive functioning of creative processes can now be characterized in reference to conceptual aspects. This will allow us to differentiate the cognitive mechanisms underlying different types of creative processes through operationalizable concept-building strategies from which general criteria for task design in education can be deduced.

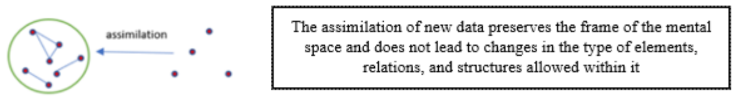
4.2. The characterization of the categories of creative processes through the theory of conceptual blending

In this section, we delve deeper into the cognitive functioning of creativity. Creative processes engage in conceptual blending strategies, while non-creative processes do not. We refer to different conceptual blending strategies to explain the distinct cognitive functioning within the three categories C1, C2, and C3 introduced in Section 4.1

Mental processes that do not involve creative thinking open a single mental space with unchanged rules and structures (Figure 3). They simply rely on cognitive organization already available in memory, effective for guiding interpretation and action. New data are assimilated preserving the frame of the mental space without changing the type of elements and their relations. In mathematics education, this is akin to exercises that reproduce formal mathematical knowledge without challenging or enriching established concepts and thinking strategies.

Figure 3

Representation of the cognitive functioning of a mental process that does not involve creative thinking: A single mental space is opened whose rules are reproduced and structures are preserved without substantial alteration

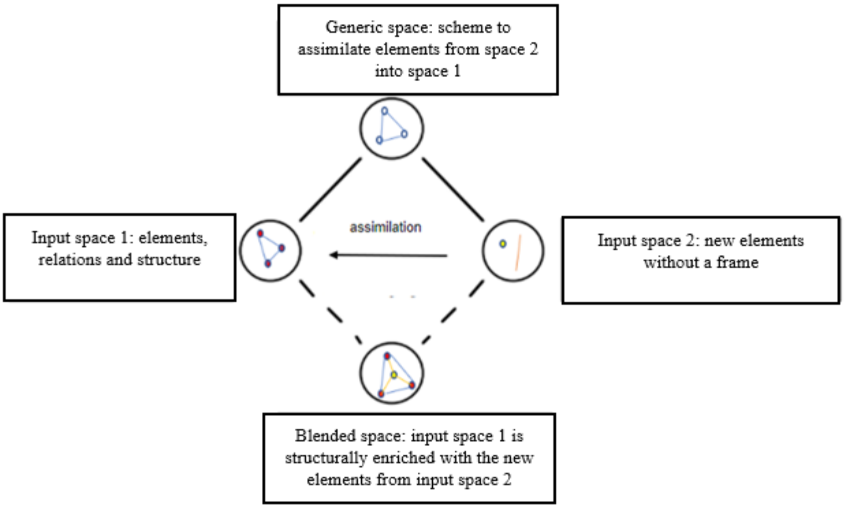


Creative processes, categorized as C1, C2, and C3, involve conceptual blending. Here, we refer to the three types of blending networks, *simplex, single-scope, and double-scope*, described in section 3.2., to frame these categories and explain their different cognitive functioning. In all these forms of conceptual blending, two mental spaces are involved as input spaces. In each of these forms, however, the relations between the two input spaces vary, and these distinctions can elucidate the differences between C1, C2, and C3 on a cognitive level, allowing also a distinction of two sub-cases for C2-creative processes.

Simplex networks explain the cognitive functioning of the first category of creative processes C1, in which originality is not accompanied by flexibility. In this case, the blending process involves the assimilation of new elements from input space 2 within the framework of input space 1 (see Figure 4).

Figure 4

Representation of the cognitive functioning of the C1-creative processes, in which originality is not accompanied by flexibility, based on the simplex network: the new external data are interpreted from the constituent rules of the initial mental space and are thus integrated within it



The resulting blending space is original compared to input space 1 because it introduces new elements and new relationships. However, it retains its original frame intact, which is not reconfigured but rather enriched and elaborated upon. The generic space represents a scheme that allows to assimilate elements from input space 2 into input space 1 without involving blending.

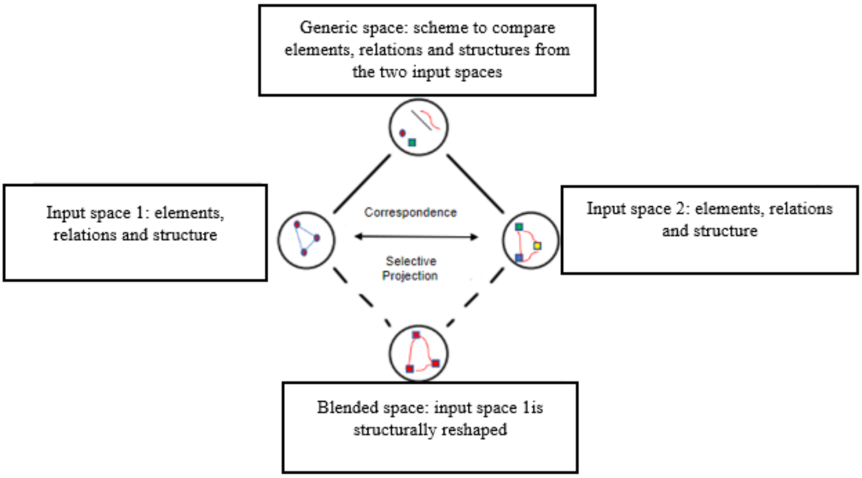
Carrying on the example discussed in section 4.1., input space 1 is rooted in the script that guides the critical evaluation of films. Within this space, numerical elements are employed to quantify judgments, serving as the foundation for numerical comparisons and relationships. When tasked with commenting on reviewers' individual judgments using provided scores, one simply engages in an exercise that does not enhance the structure of the mental space or challenge creative thinking. However, integrating different scores for the same film or comparing films with multiple scores introduces a second input space with unedited elements. While adhering to the basic rules of input space 1, this new input encourages the creative exploration of hypotheses and conceptual insights regarding numerical roles. Consequently, it allows for the introduction of new elements and relationships, such as numbers representing not only individual rankings but also averages.

Double scope networks and *single scope networks* explain the cognitive functioning of the second category of creative processes C2, allowing to distinguish two sub-categories of such processes. In both these cases, we start with two independent and unrelated input spaces, each providing a frame. The blending process that follows generates a new space that is not simply the enrichment of space 1 or 2 but is rather based on a profound qualitative transformation of the starting mental spaces.

In the case of the *double-scope networks*, input spaces 1 and 2 are brought together via analogical correspondences (i.e., recognition of similarities) between their elements, relations, and structures (see Figure 5).

Figure 5

Representation of the cognitive functioning of the first sub-category of C2- creative processes, in which originality is accompanied by flexibility, based on the double-scope network: two or more mental spaces that include unrelated elements, relations, and structures are brought together

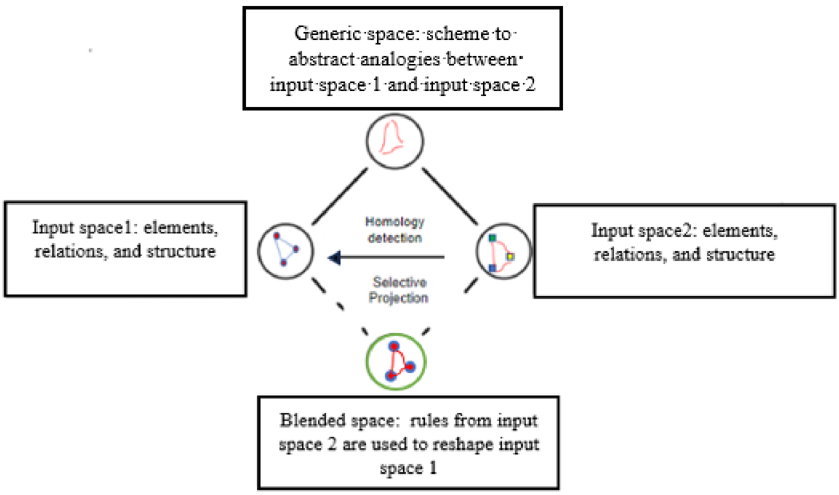


In this case the generic space represents a scheme that allows to compare elements, relations, and structures from the two input spaces.

Returning to the example of the arithmetic mean, tasks involving the visual seesaw model require more than just incorporating new data into the original script. They demand a creative and flexible effort to reinterpret it in the context of a new correlation with a distinct mental space related to balancing games on a seesaw. As a result of this divergent input, numbers are not seen only as rankings, but also as 'weights,' or 'balance', and new ways to connect and represent them are created, linked, and manipulated.

In the case of *single-scope networks*, instead, recognize elements, relations, and structures typical of one mental space as suitable for organizing other spaces. This leads to reinterpreting heterogeneous mental spaces under a common set of rules (homology detection), supporting cognitive flexibility to generalize (see Figure 6).

Figure 6
Representation of the cognitive functioning of the second sub-category of C2-creative processes, in which originality is accompanied by flexibility, based on the single-scope network: the elements, relations, and structures considered exclusive or typical of a mental space are recognized as suitable and functional for organizing other mental spaces as well



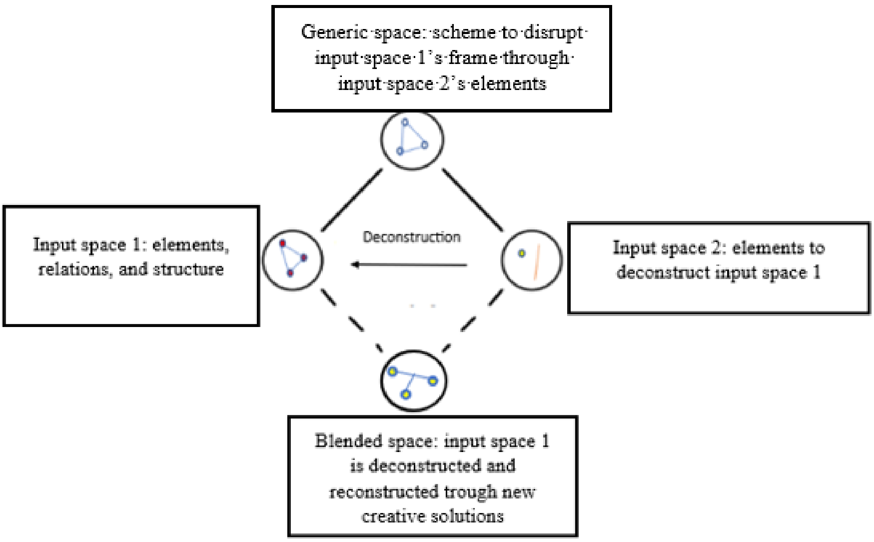
In this case the generic space represents a scheme that allows to abstract analogies between input space 2 and input space 1.

In the context of the example we are using to exemplify the cognitive functioning of the creative processes, this type of C2 process emerges, for example, when students start conversations or engage in problem-solving and problem-posing activities, in which the new conceptual insights into how numbers are used and represented are transferred to the evaluation of different films or even to completely different contexts that do not concern the critical evaluation of films in any way.

Simplex networks explain the cognitive functioning of C3-creative processes, but in a different way than they do in C1-creative processes. Indeed, the C3 category involves flexibility enabling individuals to break free from established forms of reasoning. This can be understood through a unique application of the simplex network. As introduced above, in this network, input space 1 consists of a frame, while input space 2 consists only of specific elements. Unlike C1, however, in this case blending with elements from space 2 does not enrich the frame of space 1; instead, it deconstructs it, allowing for the creation of entirely new solutions. (see Figure 7).

Figure 7

Representation of the cognitive functioning of the C3-creative processes, in which originality is accompanied by flexibility, based on simplex network: blending with a two-stage movement - a starting mental space is partially or totally deconstructed - the new input space is restructured by blending with other mental spaces - according to one of the other three types of networks

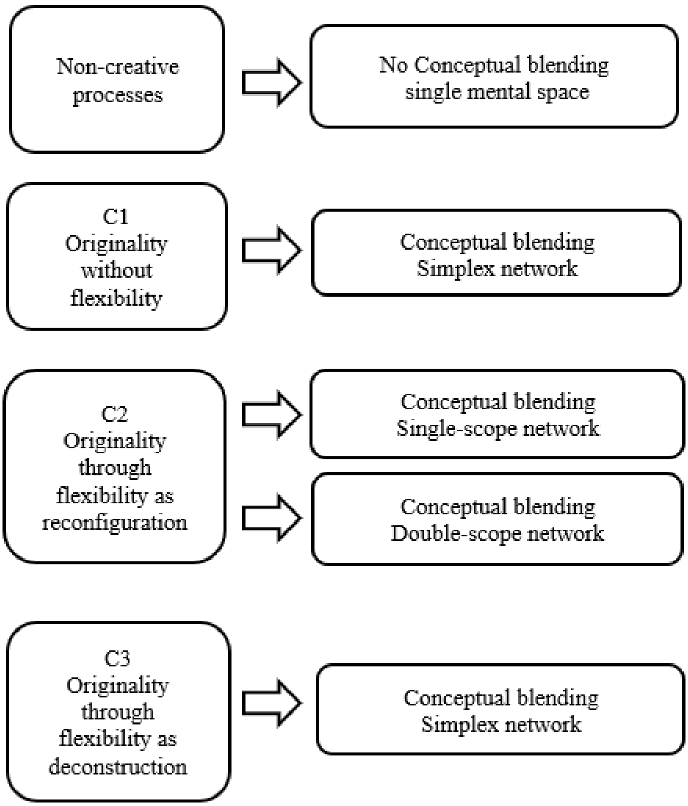


In our example, during the learning process, students engage in various C1 and C2 processes that enrich/restructure/expand the initial script regarding the numerical evaluation of films, with the creative discovery of the concept of mathematical mean and various strategies to calculate and represent it. However, they are not bound to mechanically follow the new rules of this mental space, even when they have creatively constructed them. In fact, especially when confronted with new disruptive and thought-provoking elements, they can suspend and/or alter the rules of this space to discover alternative ways to engage with ratings and averages.

Figure 8 offers a synthesis of the second step of the model, represented by the categorization of creative and not-creative processes, based on the conceptual blending networks. It shows how these networks explain the cognitive functioning of C1-,C2- and C3-processes.

Figure 8

The second step of the model: categorization of creative and non-creative processes through conceptual blending



5. General task design requirements

In this section we delineate general task design requirements to foster creativity in mathematics learning, building upon the model presented in Section 4. More specifically, in continuity with the categories of creative processes introduced above, four general types of task design requirements can be distinguished. In the following we characterize and discuss these four task design requirements.

The *first basic requirement* of task design states the importance of a mathematically significant problem defined in a context that is familiar for the student and defines a shared metal space. Suitable contexts for this *mathematically-significant-problematic-space-requirement* could be everyday experiences, narrative sources, or existing mathematical knowledge familiar to them. However, to ensure didactic significance in subsequent blending phases, the function and meaning of mathematical elements within this shared mental space must be clear and relatable. To establish this initial clarity, an extended phase of observation and dialogue with students could be necessary. A key requirement for generating this educational scenario, as emphasized in literature (Leikin & Sriraman, 2022), is presenting students with questions or problems of an appropriate degree of openness: not too open to lose relevance to the starting situation, yet not too closed to restrict creative thought. Therefore, accounting for the characteristics of the class and fostering a shared and open thinking space are vital for authentic and meaningful engagement with creativity in learning.

The *second requirement* focuses on the first category of creative processes, C1, and allows us to work on the creative insight of mathematical concepts with an *incremental-approach-requirement* that does not require flexible thinking. To enhance such creative processes, rooted in the simplex network of conceptual blending, it is crucial to introduce a new input space whose elements can mathematically enrich the initial situation. For instance, data can be presented, or thought-provoking questions can be posed, to introduce a new mathematical functionality that can be built from the initial elements and within the rules of the starting context. The conditions for this to happen depend on the teacher's ability to rethink the mathematical learning goals in the light of the starting situation and to propose them in a language consistent with the rules and structures of this shared mental space (e.g., in reference to the example used before, this could happen byframing the discovery of arithmetic mean as a problem of movies comparison). While this requirement ensures originality in thought processes, it does not explicitly prioritize flexibility. It is possible for students to spontaneously engage in more complex and flexible blending processes, but the primary focus of this requirement is not systematically fostering this aspect of creativity.

The *third requirement* affirms that tasks must require connection with elements that do not fit into the starting situation and allow students to discover and develop the divergent association independently. The main goal in this case is to work with tasks that transcend the rules of the initial situation through the association between different mental spaces. This *divergence-requirement* is related to the second category of creative processes, C2, and can be used to work with the type of cognitive flexibility based on double-scope blending or single-scope blending.

In the case of double-scope blending, task design must start from a divergent and mathematically significant interpretation of the elements encountered in the starting situation (e.g., average=balance). This premise can originate as much from the teacher's planning as from creative ideas that spontaneously emerge from students. It can be as much an incentive to develop analogical correspondences between different concepts or modes of representation (e.g., numerical calculation of the mean/verbal explanations/linear distances on the seesaw), as well as the request to manage eccentric and divergent associations (e.g., film scores - weights to balance).

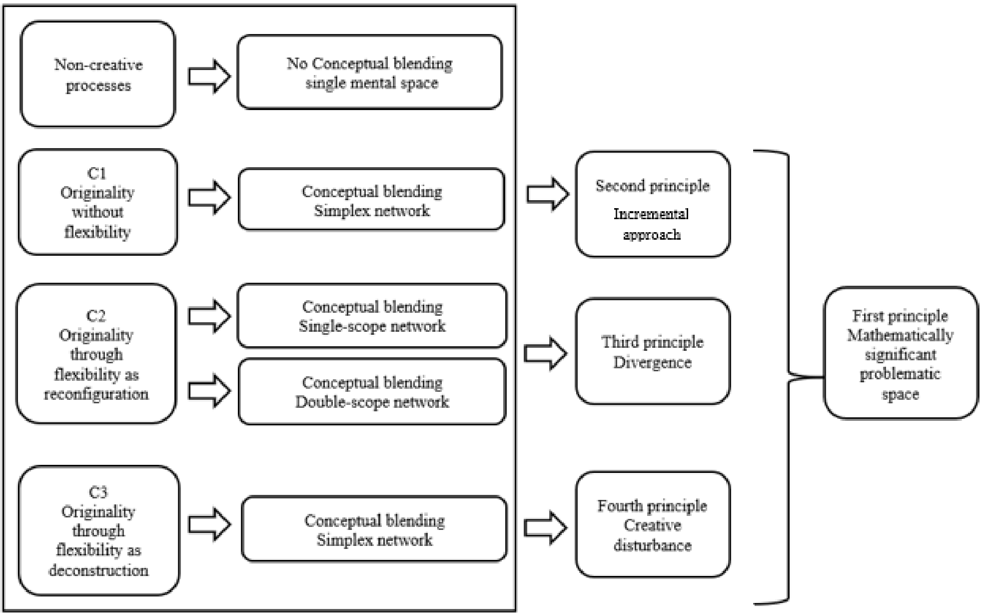
In the case of generalization through single-scope blending, instead, task design must stimulate new associations that allow for the identification of alternative contexts in which to apply what has been previously discovered (e.g., where else can we use the seesaw model to solve problems with arithmetical means?).

The *fourth requirement* is focused on the kind of flexible thinking related to simplex blending, which belongs to the third category of creative processes, C3. What is needed in this case, are creative disturbance tasks, which allow to deconstruct the starting situation that needs to be reconfigured creatively. We thus refer here to a *creative-disturbance-requirement*. Teachers can move from the simple request for alternative solutions (e.g., can we use the seesaw to find different calculation strategies?) to targeted intervention on specific elements of the initial mental space by imposing additions, elisions, or substantial modifications on properties and structures (e.g., how to communicate the average scores without writing? How can we use as few numbers as possible?).

In summary, the *mathematically-significant-problematic-space-requirement* applies to all three categories of creative processes and allows us to establish the basic conditions for working on originality and generating conceptual blending processes. The other three requirements, instead, allow us to work on additional conditions to focus on the different categories of creative processes distinguished above: the *incremental-approach-requirement* allows us to focus on category C1, by working with simplex blending and without guaranteeing cognitive flexibility; the *divergence-requirement* focuses on category, C2, by working on cognitive flexibility with double-scope and single-scope blending; the *creative-disturbance-requirement* concerns the third category, C3, and focuses on simplex blending to work on the flexible deconstruction and reconstruction of acquired strategies and ways of reasoning. Figure 9 provides an overview of the four general requirements for task design, in connection to C1, C2, C3 and the corresponding conceptual blending networks.

Figure 9

Four general requirements for task design derived from the model



6. Discussion

As a result of what has been stated so far, the model presented in section 4 provides a complete answer to the first of the two research sub-questions addressed in this study: *Which are the characteristics of a theoretical model able to provide a synthetic and comprehensive explanation of the cognitive functioning of creative processes?*

We can answer this research question by first recalling the model's components and the relations between them, and then by explaining why the model is supposed to provide a synthetic and comprehensive explanation of the cognitive functioning of creative processes.

The theoretical model presented in section 4 is based on the definition of three distinct types of creative processes, each embodying the characteristics of originality and/or flexibility in varied ways. In it, the cognitive functioning of the creative process categories defined above is clarified by the conceptual blending networks taken from the theory of conceptual blending.

The model can be considered as providing a synthetic and comprehensive explanation of the cognitive functioning of creative processes at least for three reasons.

Firstly, the model does not merely provide point-by-point descriptions of specific creative performances but provides a general classification of creative processes based on an explanation of their cognitive functioning. Consequently, in addition to empirically observing how creative thinking supports meaningful learning, with this model it is also possible to clarify how this is possible on a cognitive level.

Secondly, the model has a unified and comprehensive nature. The different types of conceptual blending networks explain how creative processes work, without depending on the punctual analysis of specific examples. Indeed, the model aims to explain all possible implementations of mini-c instances in mathematics education and, possibly, to become a framework for further descriptive research aimed to detect mini-c instances that have not yet been considered.

Finally, the explanatory potential of the model is not limited to the psychological level but has clear educational implications. The theoretical language of conceptual blending is based on concept-building strategies that can be readily operationalized, starting from elements, relations, and structures of mental spaces, up to the dynamics of combination, correlation, and reconfiguration of different input spaces within integration networks. In this sense, the model allows us to explain which tasks are suitable for stimulating creative processes that support mathematical learning.

This last point leads us to the answer to the second research sub-question: *Which task-design requirements can be deduced from this model in order to stimulate and foster creativity in mathematical learning?*

Starting from the classification and explanation of mini-c processes at the cognitive level in the previous section, we have not only confirmed the importance of open tasks, already recognized in the literature (*mathematically-significant-problematic-space-requirement*), but we have also introduced other three general design requirements to structure these tasks in a way that stimulates and develops different types of creative processes: the *incremental-approach-requirement*, needed for creative processes where no flexibility is required; the *divergence-requirement*, needed to work on flexibility in the sense of divergent associations, analogical correspondences and generalization; the *creative-disturbance-requirement*, needed to enhance flexible creative processes where fixed mental patterns are deconstructed and reshaped.

In order to answer the main research question: *Which are general requirements of task design for tasks able to stimulate and foster creativity in learning mathematics?*, we should now explain in what sense the above mentioned task design requirements can be considered as general, but also specific to mathematics education.

The task design requirements listed above can be considered as general requirements because they are independent of the specificity of the mathematical content involved in the tasks. Furthermore, they cover a wide range of possible creative cognitive behaviors that are described and characterized by basic cognitive tools, recognized in the literature as conceptual building strategies on which human thinking processes are founded. But, at the same time, these requirements are suitable for mathematics education, as we have shown with the paradigmatic example of mini-c instances, such as the creative development of conceptual insights, the manipulation and connection of representations, and the exercise of flexible thinking, taken from research in mathematics education (Pitta-Pantazzi, et al., 2022) and specific to this domain of knowledge.

7. Conclusions

Our purpose in this paper was to investigate the possibility to work out general task design requirements able to support researchers and teachers in producing suitable tasks able to stimulate and possibly develop the emergence of creative behavior within 'every day' classroom settings. To do this, we needed a theoretical model able to explain how the characteristics we included in our definition of creativity (originality and flexibility) can be linked to the cognitive strategies of conceptual blending, considered as the basic cognitive tools in human thinking (the conceptual networks single-scope, simplex, double-scope). This allowed us to 'open' what we called the 'black box' of the creativity tasks that seem to transform non-creative students into creative ones. Indeed, we were able to explain which are the cognitive processes that presumably make such tasks work, but also to which extent creative processes differ from non-creative ones. However, our investigation took a step further in operationalizing these requirements. This was accomplished by the instantiation of the characterized cognitive strategies of creative thinking by the mini-c instances known from literature in mathematics education research.

Although we were able to answer our research questions positively, our investigation also has limits: it is purely theoretical and the model we provided needs to be validated by testing the effectiveness of the derived requirements in task design. This is certainly one of the possible future research paths.

But looking up a little from the objectives set out in this paper, we believe to have also prepared the ground for future research that can take several other directions. Indeed, given the research problem we started with, the results achieved by our investigation can be considered as relevant at different levels that go beyond the ones related to the research questions.

Firstly, the requirements of task design allow for educational design in which mini-c processes are not only generically induced but become the object of explicit reflection and can be consciously linked to specific learning goals[4]. This means that further research could be carried out to provide such links, based on concrete examples of mathematical topics.

Secondly, the requirements can unify the variety of existing models and observational studies within a synthetic classification based on a shared conceptual framework provided by conceptual blending. The empirical studies focused on open problems and multiple tasks in mathematics education fall under the first requirement, which sets general didactic and pedagogical conditions for attributing a creative character to learning processes. The other three requirements, which instead add more specific details for structuring tasks, allow us to simplify the heterogenous variety of psychological theories on task design (see section 2) to three essential categories. This is not only useful for generating tasks independently but also for 'navigating' the existing literature and identifying redundancies and synergies between different theoretical approaches. We believe that this could be an important topic to be developed and discussed within the field of the epistemology of mathematics education as a research domain (Asenova, 2023).

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[4] With this, we do not mean that students' creative thinking is rigidly predictable and that the requirements of task design should be used to control it. Even when designed to reach a precise learning target (e.g., discovering the average as balance), tasks should not be conceived as prediction and control mechanisms but as conditions that catalyze the free and personal expression of creative thought (Sriraman, 2022).

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